2024

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1 – 10, write the letter associated with the correct answer.

1. The roster form of $\{x : x \text{ is a positive square root of } 4\}$ is 1

   A. $\{-2, 2\}$

   B. $\{0, 2, 4\}$

   C. $\{4\}$

   D. $\{2\}$

2. Degree measure of an angle of π radian is 1

   A. 90°

   B. 180°

   C. $\frac{180}{\pi}$

   D. 180π

3. The value of $\operatorname{cosec}(-1410^{\circ})$ is 1

A. 0

B. 1

C. 2

D. 3

4. If z is a non zero complex number then its multiplicative inverse $\frac{1}{z}$ is equal to 1

A. $\frac{\overline{z}}{|z|^2}$

B. $\frac{\overline{z}}{|z|}$

C. $\frac{z}{|\overline{z}|}$

D. $\frac{z}{|\overline{z}|^2}$

5. $1 + i + i^2 + i^3 + i^4$ is equal to 1

A. i

B. 0

C. $-i$

D. 1

6. The solution of $2 \leq 3x - 4 \leq 5$ is 1

A. $x \in (2,3)$

B. $x \in [2,3]$

C. $x \in (2,3]$

D. $x \in [2,3)$

7. If $\frac{1}{8!} + \frac{3}{7!} + \frac{x}{9!}$, then the value of x is 1

A. 225

B. 200

C. 175

D. 250

8. If $a_n = (-1)^n(2n-1)$, then the value of a_8 is 1

A. 19

B. 14

C. 15

D. 16

9. $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$ equals 1

A. 0

B. 1

C. $\frac{1}{2}$

D. 2

10. The slope of a line perpendicular to $2x-y+4=0$ is 1

A. 2

B. 1

C. -1

D. $\frac{1}{2}$

11. When are the two sets said to be equal ? 1

12. If $A \times B = \{(a, x), (a,y), (b,x), (b,y)\}$, then find A and B. 1

13. Write the value of $\sin\frac{19\pi}{4}$. 1

14. Find the value of $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$. 1

15. Define argand plane. 1

16. Express $\left(\frac{1}{3}+3i\right)^3$ in the form $a + ib$. 1

17. What is the distance between the parellel lines $Ax + By + C_1=0$ and $Ax + By + C_2 = 0$? 1

18. Define derivative of a function $f(x)$ at a given point. 1

19. State the types of measures of dispersion. 1

20. A bag contains 9 red and 12 white balls. One ball is drawn at random. Find the probability that the ball drawn is white. 1

21. Write four properties of the operation of union set. 2

22. If $\sin x=\frac{1}{3}$, find the value of $\sin 3x$. 2

23. If $\left(\frac{1+i}{1-i}\right)^m = 1$, find the least integral value of m. 2

24. Solve : $|x-2| \geq 6$. 2

25. Everybody in a room shakes hand with everybody else. If the total number of hand shakes is 66, find the total number of persons in the room. 2

26. At the end of each year the value of a certain car has depreciated 20% of its value at the beginning of that year. If its initial value was Rs. 1250, then find the value at the end of 5 years. 2

27. Find the equation of the line parellel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$. 2

28. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus. 2

29. Draw appropriate Venn diagram for each of the following : 4

(i) $A \cup B$ (ii) $A \cap B$ (iii) $A \sim B$ (iv) $B - A$

Or

Draw the graph of the function $f(x) = \frac{x^2 - 5x + 6}{x - 3}$

30. Let $A = \{x \in W : x < 2\}$, where W is whole number 4

$B = \{x \in N : 1 < x \leq 4\}$, where N is natural number

$C = \{3,5\}$

Verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Or

Find the domain and range of the function $f(x) = \sqrt{9 - x^2}$.

31. If none of the angles x, y and $x + y$ is an odd multiple of $\frac{\pi}{2}$, then prove that $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$. Hence deduce that $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$. 4

32. Find all pairs of consecutive even positive integrs, both of which are larger than 5, such that their sum is less than 23. 4

Or

IQ of a person is given by the formula $IQ = \frac{MC}{CA} \times 100$, where MA is mental age and CA is chronical age. If $80 \le IQ \le 140$ for a group of 12 years old children, find the range of their mental age.

33. A ray of light passing through the point (1,2) reflects on the x- axis at point A and the reflected ray passes through the point (5,3). Find the coordinates of A. 4

Or

If the equation of the base of an equilateral triangle is $x + y - 6 = 0$ and the opposite vertex is the point $(-1,-1)$, then find the area of the triangle.

34. Let $f(x) = \begin{cases} a + bx, x < 1 \\ 4 \quad\quad , x = 1 \\ b - ax, x > 1 \end{cases}$ and if $\lim_{x \to 1} f(x) = f(1)$, what are the possible values of a and b? 4

Or

Find the derivative of cosec x from the first principle.

35. The probability that a student will pass the final examination in both English and Mathematics is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Mathematics examination ? 4

36. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find the values of $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$. 6

Or

Prove that $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$.

37. State and prove the Binomial theorem for any positive integer n. 6

38. Find the number of different words that can be formed from the letters of the word TRIANGLE so that 6

(i) the vowels always come together

(ii) the vowels are never together

Or

A group consists of 6 boys and 4 girls. In how many ways can a committee of 5 members be formed if

(i) at least 2 girls are included

(ii) at most 2 girls are included