2023

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1– 4, write the letter associated with the correct answer.

1. The value of $\sqrt{i} + \sqrt{-i}$ is 1

   A. 0

   B. 1

   C. 2

   D. $\sqrt{2}$

2. If $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the G.M. between a and b, then the value of n is 1

   A. $\frac{1}{2}$

   B. $-\frac{1}{2}$

   C. 1

   D. –1

3. Centroid of the triangle whose vertices are (a,0,0), (0,b,0) and (0,0,c) is the point 1

A. $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

B. $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

C. (a, b, c)

D. (0, 0, 0)

4. The value of $\lim_{x\to 0}\frac{\sin ax}{bx}$ is 1

A. 0

B. 1

C. $\frac{a}{b}$

D. $\frac{b}{a}$

5. What is the difference between a function and a relation? 1

6. If A={a, b, c}, write down the power set of A. 1

7. Find the domain of the function $f(x)=\sqrt{x-9}$. 1

8. Find the value of $\sin\left(\frac{-11\pi}{3}\right)$. 1

9. If $\tan x=\frac{3}{4}$, find the value of *cos 2x*. 1

10. Find the roots of the equation $ix^2 + 2x - i = 0$. 1

11. Write down the solution set of the inequality $3x - 7 > 5x - 1$. 1

12. If P(n) is the statement '$n^2 + 2$ is divisible by 3', show that P(2) is true. 1

13. Define limit of a function $f(x)$ as x tends to a. 1

14. Write down the negation of the statement "If there is a will, then there is a way". 1

15. If $f(x) = m\frac{x-l}{m-l} + l\frac{x-m}{l-m}$, prove that $f(l) + f(m) = f(l+m)$. 2

16. The Cartesian product A × A has 9 elements among which are found (–1,0) and (0,1). Find the set A and the remaining elements of A × A. 2

17. Find the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^{12}$. 2

18. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, prove that x, y, z are in A.P. 2

19. Obtain the equation of a straight line in the intercept form. 2

20. Find the equation of the line perpendicular to the line $3x - 4y + 2 = 0$ and passing through the point (2,3). 2

21. If p is the length of the perpendicular from the origin upon the line whose intercepts on the axes are α and β, show that $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{1}{p^2}$. 2

22. Find the length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 2

23. Write down the rule for the compound statement with 'Or'. 2

24. Solve the following system of linear inequalities graphically : 4

$2x+y\le 6,\ 3x+4y\le 12,\ x\ge 0,\ y\ge 0$

25. Prove that : $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$ 4

Or

$$\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}=2$$

26. Solve : $\sin x+\sin 3x+\sin 5x=0$ 4

27. An arch is in the form of a semi-ellipse. It is 8m wide and 2m high at the centre. Find the height of the arch at a point 1.5m from one end. 4

28. If the function $f(x)$ satisfies $\lim_{x\to 1}\frac{f(x)-2}{x^2-1}=\pi$, evaluate $\lim_{x\to 1} f(x)$. 4

Or

Find the derivative of $x\sin x$ from the first principle.

29. Find the standard deviation for the following data : 4

x :	3	8	13	18	23
f :	7	10	15	10	6

30. The sum and sum of squares corresponding to length x(in cm) and weight y (in gm) of 50 plant products are given below : 4

$$\sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} x_i^2 = 902.8, \sum_{i=1}^{50} y_i = 261, \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more varying, the length or weight ?

31. Two students X and Y appeared in an examination. The probability that at least one will qualify the examination is 0.13 and both will qualify the examination is 0.02. Also probability that X will qualify the examination is 0.05. Find which one will have more chance to qualify the examination. 4

32. In any triangle ABC, prove that 6

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Or

$$a^2 = b^2 + c^2 - 2bc \cos A$$

33. Define modules and conjugate of a complex number. If z_1 and z_2 are any two complex numbers, prove that 6

(i) $z_1\bar{z}_1 = |z_1|^2$

(ii) $|z_1 z_2| = |z_1||z_2|$

(iii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $(|z_2| \neq 0)$

(iv) $\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$

34. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many people liked 6

(i) product A only.

(ii) product B only.

(iii) product C only.

35. Using the principle of mathematical induction, prove that $\forall n \in N$,

$1 + 2 + 3 + \text{---} + n < \frac{1}{8}(2n+1)^2$.

Or

$a^n - b^n$ is divisible by $a - b$, $(a \neq b)$. 6

36. There are n points in a plane of which p are collinear. Find the number of straight lines and triangles which can be formed by joining them. 6

Or

Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50^{th} word ?

37. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed. 6