2021

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1– 4, write the letter associated with the correct answer.

1. The value of $\sqrt{i}+\sqrt{-i}$ where $i=\sqrt{-1}$ 1

   A. 0

   B. 1

   C. 2

   D. $\sqrt{2}$

2. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the arithmetic mean between a and b, then the value of n is : 1

   A. 1

   B. -1

   C. $\frac{1}{2}$

   D. $-\frac{1}{2}$

3. The line segment joining the points $(-4, 8, 10)$ and $(6, 10-8)$ is divided by the YZ - plane in the ratio : 1

A. 1 : 2

B. 2 : 1

C. 2 : 3

D. 3 : 2

4. The value of $\lim_{\theta \to 0} \frac{\tan \theta^\circ}{\theta}$ is : 1

A. $\frac{180}{\pi}$

B. $\frac{\pi}{180}$

C. 1

D. 0

5. If $n(A) = p$ and $n(B) = q$, what is the number of relation from A to B ? 1

6. If $n(A) = 30$ and $n(A \cap B) = 20$, find $n(A - B)$. 1

7. If $A \times B = \{(a,1), (a,5), (1,2), (b,2), (b,5), (b,1)\}$ find $B \times A$. 1

8. Find the value of $\sin 15^\circ$. 1

9. If $\cos x = \frac{4}{5}$, find the value of $\cos 2x$. 1

10. Find the multiplicative inverse of $3-4i$. 1

11. Find the modulus of $\frac{1}{1-i}$. 1

12. Evaluate $\frac{\lfloor n}{\lfloor r \lfloor n-r}$, when $n=5$ and $r=2$. 1

13. What is meant by the derivative of a function f at the point c? 1

14. Write down the negative of the statement $p \Rightarrow q$. 1

15. Let A={1, 2, 3, ..., 14). Define a relation R from A to A by
$R=\{(x,-1): 3x-y=0, \text{ where } x, y \in R\}$. Find its domain and range. 2

16. If the function $f: R \to R$ is defined by

$$f(x)=\begin{cases} 2x+3, & \text{if } x<-2 \\ x^2-2, & \text{if } -2 \le x \le 3 \\ 3x-1, & \text{if } x>3 \end{cases}$$

find $f(2), f(4), f(-1)$ and $f(-3)$. 2

17. Find the coefficient of x^9 in the expansion of $\left(x^2-\frac{1}{x}\right)^9$. 2

18. Fnd the sum of the following series up to n terms :

$5 + 55 + 555 + ...$ 2

19. P (a,b) is the mid-point of a line segment between the axes. Show that the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$. 2

20. If p is the length of the perpendicular from the origin to the line where intercepts on the axes are a and b, show that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$. 2

21. Write down the expression for the acute angle θ between the lines whose slopes are m_1 and m_2. Hence obtain the condition for perpendicularity of the lines. 2

22. What is meant by the eccentricity of an ellipse ? Write down the expression for the eccentricity e of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 2

23. Define a conditional statement and its contrapositive. 2

24. Prove that : $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=\frac{1}{8}$ 4

OR

Prove that: $\sin\frac{\pi}{9}\ \sin\frac{2\pi}{9}\ \sin\frac{3\pi}{9}\ \sin\frac{4\pi}{9}=\frac{3}{16}$.

25. Prove that $\sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y$ and hence deduce that $\cos^2 2x-\cos^2 6x=\sin 4x\ \sin 8x$. 4

OR

If $\cos(x-y)+\cos(y-z)+\cos(z-x)=-\frac{3}{2}$,

prove that $\cos x+\cos y+\cos z=\sin x+\sin y+\sin z=0$

26. Solve the following system of inequalities graphically: 4

$x + 2y \leq 8$

$2x + y \leq 8$

$x \geq 0$

$y \geq 0$.

27. Obtain the equation of a parabola in the form $y^2 = 4ax$. 4

28. If the function $f(x)$ satisfies $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \to 1} f(x)$. 4

29. Find the standard deviation of the following data : 4

x_i :	3	8	13	18	23
f_i :	7	10	15	10	6

30. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below :

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40·9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks ? 4

31. Out of 120 students, two sections of 50 and 70 are formed. If you and your friend are among the 120 students, find which is more likely to happen that you both enter the same section or different sections. 4

32. Find the sum of the cubes of the first n natural numbers. 6

33. Derive the expression for $\cos(x + y)$ in terms of sines and cosines of x and y. 6

OR

Prove that $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$ and hence deduce the expressions for $\tan(x-y)$ and $\tan 2x$.

34. In a survey it was found that 21 people liked product A, 26 people liked product B and 29 liked product C. If 14 people liked products A and B, 14 people liked products B and C, 12 people liked products C and A and 8 liked all the three products. Find how many liked product C only. 6

35. If the roots of the equation $mx^2 + nx + n = 0$ in the ratio $l : k$, show that

$$\sqrt{\frac{l}{n}} + \sqrt{\frac{n}{l}} - \sqrt{\frac{n}{m}} = 0. \qquad 6$$

OR

If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\dfrac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$.

36. Derive the formula ${}^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \le r \le n$ for the number of permutation of n different objects taken r at a time. 6

37. How many triangles can be formed by joining 10 points of which 5 points are in the same straight line ? Find also the number of lines formed by joining them. 6

OR

Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50^{th} word ?
