2020

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1–4, write the letter associated with the correct answer.

1. The roster form of the set $S = \{x / x^2 = x, x \in \mathbb{R}\}$ is –

   A. $\{0\}$

   B. $\{-1\}$

   C. $\{0, 1\}$

   D. $\{1\}$ 1

2. Cot x is defined for all x in –

   A. R

   B. $R - \{n\pi : n \in I\}$

   C. $R - \{(2n+1)\frac{\pi}{2} : n \in I\}$

   D. $R - \{(2n-1)\frac{\pi}{2} : n \in I\}$ 1

3. The term containing x^3 in the expansion of $(x-2y)^7$ is –

A. 5^{th}

B. 4^{th}

C. 3^{rd}

D. 2^{nd} 1

4. The foci of a hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ are –

A. $(\pm a, 0)$

B. $(0, \pm a)$

C. $(\pm ae, 0)$

D. $(0, \pm ae)$ 1

5. Define power set. 1

6. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x^2 + 3$ is not one-one. 1

7. Find the value of 1

$2 \sin 15° \cos 15°$.

8. Write the multiplicative inverse of $\sqrt{5} + 3i$. 1

9. State the principle of mathematical induction. 1

10. Find the number of terms in the expansion of $(1 + 2x + x^2)^{11}$. 1

11. Write the number of integers between 100 and 1000 that are divisible by 7. 1

12. Define the derivative of a function $f(x)$ at any point x, in its domain. 1

13. Find $\frac{dy}{dx}$ if $y = \sin^2 2x$ at $x = \frac{\pi}{8}$. 1

14. What is meant by a compound statement ? 1

15. Express the complex number $x + iy$ in polar form. 2

16. In an experiment, an acid solution is kept between 40°C and 45°C . What is the range of temperature in degree Fahrenheit, if the conversion formula is given by $C = \frac{5}{9}(F - 32)$, where C and F represent temperature in degree Celsius and degree in Fahrenheit respectively. 2

17. Write the middle terms in the expansion of $(x + a)^n$, $n \in N$. 2

18. Let $P(n)$ be a statement "$2^{3n} - 1$ is divisible by 7". Prove that if $P(m)$ is true then $P(m + 1)$ is also true, $m \in N$. 2

19. If a polygon has 44 diagonals, then find the number of its sides. 2

20. 100 stones are placed on a straight road at intervals of 5 m apart. A runner has to start from a fixed point 5 m from the first stone in the same straight line, pick up the stones and bring them back to the fixed point one by one. How many kilometer has he to run altogether ? 2

21. Find the equation of a straight line passing through two given points (x_1, y_1) and (x_2, y_2). 2

22. The focus of a parabolic mirror is at a distance of 4 cm from the vertex. If the mirror is 16 cm deep, find the diameter of the mirror. 2

23. If p and q are two statements given by 2

p : 25 is a multiple of 5.

q : 25 is a multiple of 8

Write the compound statement by using the connective '*and*' and '*OR*' check its validity.

24. Draw the graph of the function : 4

$$f(x) = \frac{x^2 - x - 6}{x + 2}$$

(Don't use graph paper)

25. In any ΔABC, prove that 4

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

26. Solve : $\sin\theta + \sin 3\theta + \sin 5\theta = 0$ 4

27. Find the square root of $1 + i$. 4

28. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second ? 4

29. Find the coordinates of the foot of perpendicular from the point (–2, 3) to the line $2x - y = 3$ and also the length of perpendicular from the given point to the line. 4

30. Define a circle. Derive the general equation of a circle. 4

OR

Find the coordinates of the point which divides the line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ internally.

31. Evaluate : $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$. 4

OR

If $y = \frac{\cos x}{1 + \sin x}$, then find $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$.

32. In a survey of 200 people, it was found that 40 people like food A, 25 like food B and 15 like food C. If 10 people like foods A and B, 6 like foods B and C, 8 like foods A and C and 3 like foods A, B, C, find the number of people who like

(i) Food A only

(ii) Food B only

(iii) Food C only 6

33. (i) Prove that $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$.

(ii) Prove that $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\left(\frac{x+y}{2}\right)$. 3+3=6

OR

If $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$ where α, β lie between 0 and $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$. 6

34. How many permutations can be formed by using the letters of the word 'TRIANGLE' when 6

(i) there is no restriction on letters ?

(ii) each word begins with A ?

(iii) each word begins with R and ends with G ?

(iv) all the vowels come together ?

(v) all the consonants come together ?

OR

A committee of 7 has to be formed from 9 gentlemen and 4 ladies. In how many ways can this be done when the committee consist of –

(i) exactly 3 ladies ?

(ii) at least 3 ladies ?

(iii) at most 3 ladies ?

35. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. prove that

$(q + p) : (q - p) = 17 : 15$ 6

36. Find the mean deviation about median for the following data : 6

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of Girls	6	8	14	16	4	2

OR

Find the mean, variance and standard deviation of the following data :

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

37. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among 100 students, then what is the probability that 6

(i) You both enter the same section ?

(ii) You both enter the different section ?