2025

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1–10, write the letter associated with the correct answer.

1. $\int e^x(\sin x+\cos x)dx$ equals 1

(A) $e^x \sin x+C$ (B) $e^x \cos x+C$

(C) $-e^x \sin x+C$ (D) $-e^x \cos x+C$

2. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is 1

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{12}$

(C) $\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

3. If $x=a^{\sin^{-1}t}$ and $y=a^{\cos^{-1}t}$, then $\frac{dy}{dx}$ equals 1

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$

(C) $-\frac{x}{y}$ (D) $-\frac{y}{x}$

4. If $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$, then the value of x is 1

(A) 2 (B) 1

(C) 0 (D) −1

5. The interval in which $f(x) = x^2e^{-x}$ is increasing in 1

(A) $(-\infty,\infty)$ (B) $(-2,0)$

(C) $(2,\infty)$ (D) $(0,2)$

6. The rate of change of the area of circle with respect to its redius r at r = 6 cm is 1

(A) 6π cm² / cm (B) 6π cm / cm²

(C) 12π cm² / cm (D) 12π cm / cm²

7. The vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{2}{3}$. Then $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is 1

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

8. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is 1

(A) $e^x + e^y = C$ (B) $e^{-x} + e^y = C$

(C) $e^x + e^{-y} = C$ (D) $e^{-x} + e^{-y} = C$

9. If $\propto, \beta, \gamma$ are the angles made by a line with the cordinate axes, then $\cos 2\propto + \cos 2\beta + \cos 2\gamma$ equals 1

(A) 1 (B) 2

(C) –1 (D) –2

10. The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{k}\times\hat{i})+\hat{k}.(\hat{i}\times\hat{j})$ is 1

(A) 0 (B) 1

(C) 2 (D) 3

11. If $f:\{1, 3, 4\}\rightarrow\{1, 2, 5\}$ and $g:\{1, 2, 5\}\rightarrow\{1, 3\}$ be given by

$f=\{(1, 2), (3, 5), (4, 1)\}$ *and* $g=\{(1, 3), (2, 3), (5, 1)\}$, write down gof. 1

12. Find the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. 1

13. Write down the domain of $\operatorname{cosec}^{-1}$. 1

14. If A and B are symmetric matrices, prove that AB – BA is a skew-symmetric matrix. 1

15. If $x=f(t)$ and $y=g(t)$, find $\frac{dy}{dx}$. 1

16. Find $\frac{dy}{dx}$, if $y=\sqrt{\sin(ax+b)}$ 1

17. Evaluate : $\int_{2}^{\sqrt{3}}\frac{xdx}{x^2+1}$ 1

18. What is meant by a particular solution of a differential equation ? 1

19. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$ 1

20. When is a function $f(x)$ said to be differentable at $x = a$? 1

21. Prove that the function f defined by $f(x)=\begin{cases} \frac{\sin x}{x}, if\ x<0 \\ x+1, if\ x\geq 0 \end{cases}$ is continuous at $x=0$ 2

22. Solve : $\frac{dy}{dx}+y=1,(y\neq 1)$ 2

23. Find the distance between the parallel line

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=(2\hat{j}+4\hat{j}+5\hat{k})+\mu(2\hat{i}+3\hat{j}+4\hat{k})$. 2

24. Solve : $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$ 2

25. Define a linear programming problem (LPP). What is meant by the objective function of a LPP ? 2

26. Prove that : $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+x^2}\right)\right]=\frac{x+y}{1-xy}$. 2

27. Reduce the equation $(\tan^{-1} y-x)\, dy=(1+y^2)dx$ in the linear form and hence obtain the integrating factor of the equation. 2

28. Find the value of p so that the line

$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. 2

29. (a) Show that the relation defined in the set A of all triangles as $R=\{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ is equivalence relation. Consider three right angle triangles T_1 with sides 5, 12, 13, T_2 with sides 9, 12, 15 and T_3 with sides 15, 20, 25. Which triangles among T_1, T_2 and T_3 are related? 4

OR

(b) Let $f: N \to Y$ be a function defined as $f(x) = 4x+3$, when $Y = \{y \in N : y = 4x+3 \text{ for some } x \in N\}$. Show that f is invertible. Find also the inverse of f.

30. (a) If $y = e^{a \cos^{-1} x}, (-1 \leq x \leq 1)$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$. 4

OR

(b) If $(\cos x)^y = (\cos y)^x$, then prove that $\frac{dy}{dx} = \frac{\log(\cos y) + y \tan x}{\log(\cos x) + x \tan y}$.

31. (a) Prove that $\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2-a^2} - a^2 \log\left|x + \sqrt{x^2-a^2}\right|\right] + C$ 4

OR

(b) State and prove the formula for integration by parts.

32. Solve the following LPP graphically :

Miximise $Z = x + 2y$ subject to the constrains. 4

$2x + y \geq 3$
$x + 2y \geq 6$
$x, y \geq 0$

(Graph paper will not be supplied)

33. Find by integration the area of the region bounded by the parabola $y^2 = 4x$ the line $x = 4$. 4

34. (a) Derive the vector equation of a line passing through a given point and parallel to a given vector. 4

OR

(b) Find the shortest distance between the skew lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and

$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$.

35. (a) Given that the events A and B are such that

$P(A) = \frac{1}{2}, P(B) = p \text{ and } P(AUB) = \frac{3}{5}$. Find p if they are (i) mutually exclusive (ii) independent. 4

OR

(b) Probability of solving a specific problem independently by A and B are $\frac{1}{7}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exectly one of them solves the problem.

36. Define adjoint of a square matrix A. Prove that $A(adj) = (adj)A = |A| I$, where I is the identity matrix of the same order as A and deduce that if A is a non - singular matrix, then $A^{-1} = \frac{1}{|A|}(adj\, A)$. 6

37. A manufacturer produces three products *x, y, z* which he sells in two markets. Annual sales are indicated below :

Market	Produces (in numbers)		
	x	y	z
A	10000	2000	18000
B	6000	20000	8000

(i) If unit sale prices of x, y, z are Rs. 5, Rs. 3 and Rs. 2 respectively, find the total revenue in each market with the help of matrix algebra.

(ii) If the unit costs of the above three commodities are Rs. 4, Rs. 2 and Re. 1 repectively, find the gross profit in each market with the help of matrix algabra. 2+4=6

38. (a) A B is a diameter of a circle and C is any point on the circle. Show that the area of triangle ABC is maximum when it is isosceles. 6

OR

(b) Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given right circular cone is half that of the cone.

39. (a) Prove by using vector that the median to the base of an isosceles triangle is perpendicular to the base. 6

OR

(b) If the vertices A, B, C of a triangle ABC are (1, 1 ,2), (2, 1, –1), (–1, 1, 2) respectivetly, find sin A by vector method.

40. (a) Evaluate : $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$ 6

OR

(b) Show that : $\int_0^1 \cot^{-1}(1-x+x^2)dx = \frac{\pi}{2} - \log 2$

41. (a) Bag I contains 4 red and 5 black balls and Bag II contains 3 red and 4 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be red in colour. Find which is more probable that the transferred ball is red or black. 6

OR

(b) A man is known to speak the truth 3 times out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is actually greater than 4 or not greater than 4.