2025
MATHEMATICS

Full Marks : 100
Pass Marks : 33
Time : Three Hours

Attempt all Questions

The figures in the right margin indicate full marks for the questions.
For Question Nos. 1-10, write the letter associated with the correct answer.

1. Let R be a relation in the set N given by 1

   $R=\{(a,b): a=b-2, b>6\}$

   Then, the correct option is

   (A) $(8,7)\in R$ (B) $(6,8)\in R$

   (C) $(3,8)\in R$ (D) $(2,4)\in R$

2. If $A=\begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2=I$, then 1

   (A) $1+a^2+bc=0$ (B) $1-a^2+bc=0$

   (C) $1-a^2-bc=0$ (D) $1+a^2-bc=0$

3. If $y=Ae^{5x}+Be^{-5x}$, then $\dfrac{d^2y}{dx^2}$ is equal to 1

   (A) $25y$ (B) $50y$

   (C) $-25y$ (D) $15y$

4. The rate of change of the area of a circle with respect to it radius r at r = 6 cm is 1

   (A) 10π (B) 12π

   (C) 8π (D) 11π

5. $\int e^{5\log x}dx$ is equal to 1

(A) $\frac{x^5}{5}+C$ (B) $\frac{x^6}{6}+C$

(C) $5x^4+C$ (D) $6x^5+C$

6. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ is 1

(A) 4 (B) $\frac{3}{2}$

(C) 2 (D) 3

7. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0, then the angle between $\vec{a}$ and $\vec{b}$ is 1

(A) $\frac{\pi}{2}$ (B) π

(C) $\frac{\pi}{4}$ (D) 0

8. If two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel then 1

(A) $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{c_1}{c_2}$ (B) $\frac{a_1}{a_2}=\frac{b_2}{b_1}=\frac{c_2}{c_1}$

(C) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_2}{c_1}$

9. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{k}$ and $\frac{x-1}{k}=\frac{4-y}{-2}=\frac{z-5}{-2}$ are mutually perpendicular, if the value of k is 1

(A) $\frac{-2}{3}$ (B) $\frac{2}{3}$

(C) -2 (D) 2

10. The corner points of the feasible region of a LPP are (0,4), (8,0) & $\left(\frac{20}{3},\frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then the difference between maximum value of Z and minimum value of Z is equal to 1

(A) 144 (B) 96

(C) 120 (D) 136

11. What is the range of the principal value branch of the function $cosec^{-1}$? 1

12. Define a diagonal matrix. 1

13. If matrix $A=[0,1,2]$, write AA' where A' is the transpose of matrix A. 1

14. Define derivative of a function f at the point c in its domain. 1

15. Define an increasing function. 1

16. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x dx$. 1

17. What is meant by the general solution of a differential equation ? 1

18. Define scalar product of two vectors. 1

19. Write the vector equation of a line passing through the point (1,–1,2) and parallel to the line whose equation is 1

$$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$$

20. Define feasible region of a LPP. 1

21. Given the relation R ={(1,1), (2,3), (1,2)} on the set A= {1,2,3}, add a minimum number of ordered pairs so that the enlarged relation R is reflexive and transitive. 2

22. Show that the signum function $f: R \to R$, given by 2

$$f(x)=\begin{cases} 1, & \text{if } x>0 \\ 0, & \text{if } x=0 \\ -1, & \text{if } x<0 \end{cases}$$

is neither one one nor onto

23. Differentiate x^6 w.r.t. $\log x$ 2

24. Solve the differential equation

$$\cos\left(\frac{dy}{dx}\right)=a, \qquad (a \in R)$$ 2

25. If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then prove that its area $=\frac{1}{2}|\vec{a}\times\vec{b}|$. 2

26. Find the angle between the following pair of lines

$\vec{r}=\hat{i}-4\hat{j}+\lambda(\hat{i}+2\hat{j}-2\hat{k})$ and $\vec{r}=\hat{i}+\hat{j}-2\hat{k}+\mu(2\hat{i}+4\hat{j}-4\hat{k})$ 2

27. If E and F are events of a sample space S of an experiment, then prove that

P(E'/F) = 1 – P(E/F) 2

28. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red. 2

29. Prove that : $\tan^{-1}\left(\frac{63}{16}\right)=\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$ 4

30. (a) Prove that every square matrix is uniquely expressed as the sum of a symmetric matrix and a skew - symmetric matrix. 4

Or

(b) If the inverse of a square matrix exists, prove that it is unique. If A and B are both invertible square matrices of the same order, prove that $(AB)^{-1}=B^{-1}A^{-1}$

31. (a) Find the area of the region bounded by the curve $x^2+y^2=16$ and lines $y=1$ & $y=2$. 4

Or

(b) Find the area of the region bounded by the curve $y^2=4x$, y - axis and the line $y=3$.

32. (a) Solve the following differential equation 4

$xdy-ydx=\sqrt{x^2+y^2}\,dx$, given that $y=0$ when $x=1$

Or

(b) Solve the differential equation

$\frac{dy}{dx}+y\cot x=2\cos x$, given that $y=0$ when $x=\frac{\pi}{2}$.

33. (a) Show that the points A,B,C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively are the vertices of a right angled triangle. Hence, find the area of the triangle. 4

Or

(b) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, then find a vector $\vec{c}$, such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.

34. (a) Derive the vector equation of a line passing through a given point and parallel to a given vector and hence obtain the cartesian equation of the line. 4

Or

(b) Prove that the shortest distance between two parallel lines $\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$ is

$$\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

35. Solve the following LPP graphically : 4

Maximize and Minimize $Z=3x+4y$

Subject to the constraints

$x+2y\leq 10$

$2x+y\leq 14$

$x,y\geq 0$

(Graph paper will not be supplied)

36. Solve the following system of linear equations using matrix method. 6

$x+2y+3z=6$

$2x-y+z=2$

$3x+2y-2z=3$

37. Find the values of p and q for which 6

$$f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & if\ x<\dfrac{\pi}{2}\\ p, & if\ x=\dfrac{\pi}{2}\\ q\dfrac{(1-\sin x)}{(\pi-2x)^2}, & if\ x>\dfrac{\pi}{2}\end{cases}$$

is continuous at $x=\dfrac{\pi}{2}$

38. (a) The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is the diameter of the circle. 6

Or

(b) Find the volume of the largest cylinder that can be inscribed in a sphere of radius R.

39. (a) Prove that $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$ 6

Or

(b) Evaluate : $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$

40. (a) Evaluate : $\int_0^{\pi/2} \log\cos x\,dx$ 6

Or

(b) Prove that $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}$

41. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ? 6

Or

A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is 2. Find the probability that it is actually 2 or it is not actually 2.