2020

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1-4, write the letter associated with the correct answer.

1. The identity element of the binary operation $*$ on R defined by $a * b = \frac{ab}{4} \ \forall \ a, b \in R$ is :

A. 0

B. 1

C. 4

D. 16 1

2. The value of $\sin\left(2\tan^{-1}\frac{1}{3}\right)$ is :

A. $\frac{4}{5}$

B. $\frac{3}{5}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$ 1

3. The degree of the differential equation

$$\frac{d^3y}{dx^3} + 2x^3\left(\frac{d^2y}{dx^2}\right)^2 + 3x^2\left(\frac{dy}{dx}\right)^3 + y = 0 \text{ is}$$

A. 1

B. 2

C. 3

D. 4 1

4. The distance between the planes $2x + 3y + 6z + 5 = 0$ and $2x + 3y + 6z = 9$ is :

A. 14 units

B. 7 units

C. 4 units

D. 2 units 1

5. What is meant by an equivalence relation? 1

6. For any square matrix A, prove that AA' is a symmetric matrix. 1

7. Find the value of k for which the function

$$f(x) = \begin{cases} \frac{\sin 3x}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$$

is continuous at $x = 0$. 1

8. If $y = a \sin 3x - b \cos 3x$, find $\frac{d^2y}{dx^2}$ in terms of y. 1

9. Let R be a relation in N defined by a R b if 5 divides $a - b$, (a, b∈N). Prove that $a\,R\,b \Rightarrow a^2\,R\,b^2$. 1

10. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$. 1

11. If $f(x) = \int_0^x t \sin t\, dt$, write down the value of $f'(x)$. 1

12. What is meant by a linear differential equation of the first order ? 1

13. Define derivative of a function *f(x)* at the point $x = 0$. 1

14. State chain rule for finding the derivative of a composite function. 1

15. Find the inverse of the function $f : R \longrightarrow R$ defined by

$f(x) = \frac{3x+1}{2}\ \forall x \in R$. 2

16. State Lagrange's Mean Value Theorem and give its geometrical interpretation. 2

17. Evaluate : $\int_0^8 |x-5|\,dx$ 2

18. If $\vec{a}$ is any vector in space, show that

$\vec{a} = (\vec{a}.\hat{i})\,\hat{i} + (\vec{a}.\hat{j})\,\hat{j} + (\vec{a}.\hat{k})\,\hat{k}$ 2

19. What is meant by a homogeneous differential equation ? Describe how it can be reduced to a form in which the variables are separable. 2

20. Evaluate: $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ 2

21. Find the least value of k such that the function $f(x) = x^2 + kx + 1$ is increasing in the interval $(1, 2)$. 2

22. Find the equation of the plane through the point $(1, -1, 3)$ and parallel to the plane $\vec{r}.\,(2\hat{i} + 3\hat{j} - 4\hat{k}) + 5 = 0$. 2

23. Find the distance between the two parallel lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda\,(2\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu\,(2\hat{i} + \hat{j} + \hat{k})$ 2

24. If $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$. 4

25. If $x^a y^b = (x+y)^{a+b}$, prove that $\frac{dy}{dx} = \frac{y}{x}$, provided $ay \neq bx$. 4

OR

If the function f defined by

$$f(x) = \begin{cases} x^2, & x \leq c \\ ax + b, & x > c \end{cases}$$

in differentiable at $x = c$, find the values of a and b.

26. Prove that $\int \sqrt{x^2+a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2 \log|x+\sqrt{x^2+a^2}|\right] + c.$ 4

27. Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx.$ 4

28. Find, by integration, the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum. 4

OR

Find, by integration, the area of the region bounded by the curves $y^2 = 4x$ and $x^2 = 4y$.

29. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is $(x+y+1) = 4\ (1-x-y-2xy)$, where A is an arbitrary constant. 4

30. Obtain the equation of a plane in the intercept form. 4

31. What is meant by a random variable ? Define its probability distribution, mean and variance. 4

32. If $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, show that $A^3 - A^2 - 3A - I = 0$ and hence find A^{-1}.

6

33. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. 6

OR

Find the point on the curve $x^2 = 2y$ which is nearest to the point (0, 5).

34. Solve the following system of equations by matrix method : 6

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2.$$

35. Prove by vector method that the internal bisectors of the angles of a triangle are concurrent. 6

OR

Prove by vector method that in any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} .$$

36. In a bolt factory, machines A, B and C manufacture respectively 60% , 25% and 15% of the total bolts rexpectively. Of the total of their output 1%, 2% and 1% are defective bolts. A bolt is drawn at random from the total production and found to be defective. From which machine, the defective bolt in expected to have been manufactured ? 6

OR

A box contains 10 items out of which 2 are defective. A man selects 3 items at random. Find the expected number of defective items he has drawn.

37. An industry manufactures toy cars and cycles. It can invest Rs. 8900 in both of them. A car costs Rs. 450 and a cycle costs Rs. 350. It has a storage capacity of 22 items only. If its profit in Rs. 60 per car and Rs. 50 per cycle, how many of each should be manufactured so that the profit is maximum? Find also the maximum profit graphically. 6
