2019

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

*For Question Nos. **1 – 6,** write the letter associated with the correct answer.*

1. The value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is :

   A. 1

   B. $\frac{1}{2}$

   C. $\frac{1}{\sqrt{2}}$

   D. 0 1

2. If $f : R \longrightarrow R$ be given by $f(x) = (3-x^3)^{\frac{1}{3}}$, then $f^{-1}(x)$ equals :

   A. x^3

   B. $x^{\frac{1}{3}}$

   C. $3-x^3$

   D. $(3-x^3)^{\frac{1}{3}}$ 1

3. Mean and variance of a binomial distribution are 12 and 3 respectively. Then the number of trials is :

A. 12

B. 15

C. 16

D. 36 1

4. $\int e^x \sec x\,(1+\tan x)dx$ equals :

A. $e^x \cos x + C$

B. $e^x \sec x + C$

C. $e^x \sin x + C$

D. $e^x \tan x + C$ 1

5. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is :

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$ 1

6. If the line $\vec{r} = (-2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(-K\hat{i} + 2\hat{j} + \hat{k})$ is parallel to the plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) + 7 = 0$, then the value of K is :

A. 0

B. 1

C. -1

D. -2 1

7. Show that the operation * on R_+ (set of all positive real numbers) defined by $a * b = \frac{ab}{3}$, $\forall a, b \in R_+$ is a binary operation on R_+. 1

8. Is Rolle's Theorem applicable to the function $f(x) = |x|$ in the interval $[-1, 1]$? 1

9. If $\frac{dy}{dx} = \frac{y}{x}$, prove that $\frac{d^2y}{dx^2} = 0$. 1

10. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 5$ is increasing in R. 1

11. Evaluate : $\int_1^{\sqrt{3}} \frac{1}{1+x^2}\, dx$. 1

12. What is meant by the general solution of a differential equation ? 1

13. If $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, find the angle between $\vec{a}$ and $\vec{b}$. 1

14. Define position vector of a point. 1

15. The certesian equation of a line is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Write its vector form. 1

16. If α, β, γ be the angles made by a line with the coordinate axes, prove that

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. 1

17. Show that the relation R on $N \times N$ defined by

$(a,b)\ R\ (c,d) \Leftrightarrow a+d=b+c,\ \forall\ (a,b),\ (c,d)\in N\times N$

is an equivalence relation. 3

18. If $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2,

show that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. 3

19. Evaluate $\int_0^{\pi/2}\frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$. 3

20. Prove that $\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+C$. 3

21. Find the differential equation of the family of curves $y=e^x(A\cos x+B\sin x)$, where A and B are arbitrary constants. 3

22. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces. 3

23. Prove that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), (xy < 1)$ and hence deduce that

(i) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), (xy > -1)$

(ii) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), (|x| < 1)$ 4

24. If the inverse of a square matrix exists, prove that it is unique. If A and B are both invertible square metrices of the same order, prove that $(AB)^{-1} = B^{-1}A^{-1}$. 4

25. If $f(x)$ defined by

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , \text{If } x < 0 \\ c & , \text{If } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & , \text{If } x > 0 \end{cases}$$

is continuous $at\, x = 0$, find the values of a, b and c. 4

26. Find $\frac{dy}{dx}$, if $x^y + y^x = a^b$. 4

OR

If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(\log ex)^2}$.

27. Find the area of the region bounded by the triangle whose vertices are $(-1, 2)$, $(1, 5)$ and $(3, 4)$. 4

OR

Find the area of the region bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

28. Find the integrating factor of the linear equation $\frac{dy}{dx} + Py = Q$ and hence obtain the general solution of the equation. 4

29. Define cross product of two vectors and give the geometrical interpretation of the cross product of two vectors. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, obtain the algebraic formula for $\vec{a} \times \vec{b}$. 6

30. Prove that : 6

$$\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2} = \frac{\pi}{1 - a^2};\ (a < 1)$$

OR

$$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}.$$

31. State and prove Baye's Theorem. 6

32. Derive the vector equation of a line passing through a given point and parallel to a given vector and hence obtain the cartesian equation of the line. 6

OR

Derive the vector equation of a plane in the normal form and hence obtain the cartesian equation of the plane.

33. Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$. 6

OR

Prove that the curves $y^2 = x$ and $xy = k$ cut at right angle if $8k^2 = 1$.

34. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB and hence solve the following system of linear equations :

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7.$$

6

35. Two godowns A and B have a given storage capacity of 100 quintals and 50 quintals respectively. They supply grain to three ration shops D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from godowns to the shops are given in the table below :

To / From	Transportation cost per quintal (in rupees)		
	D	E	F
A	6	3	2·50
B	4	2	3

How should the supplies be transported in order that the transportation cost is minimum ? Solve the problem graphically. 6