2024

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1 – 10, write the letter associated with the correct answer.

1. Let $A = \{1,2,3\}$. The number of equivalence relations containing $(1,2)$ is 1

   A. 4 B. 3

   C. 2 D. 1

2. The range of $\sec^{-1}$ (principal value branch) is 1

   A. $[0,\pi]$ B. $(0,\pi)$

   C. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ D. $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

3. The principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ 1

   A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$

   C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$

4. If A is a matrix of order 2×3 and B is a matrix of order 3×4, then the order of $(AB)'$ is 1

A. 2×3

B. 2×4

C. 4×2

D. 3×4

5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is the Co-factors of a_{ij}, then the value of Δ is given by 1

A. $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

B. $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

C. $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

D. $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

6. If the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is a singular, then the value of x is 1

A. 2

B. 1

C. 0

D. –1

7. If $x = at^2, y = 2at$, then $\frac{dy}{dx}$ is equal to 1

A. $\frac{1}{t}$

B. $-t$

C. t

D. $-\frac{1}{t}$

8. A general solution to a differential equation is that in which the number of arbitrary constants is 1

A. equal to the degree of the equation

B. more than the order of the equation

C. equal to the order of the equation

D. less than the order of the equation

9. A line makes an angle 90°, 60° and θ with x, y and z axis respectively, where θ is acute, then the value of θ is 1

A. 90°

B. 60°

C. 45°

D. 30°

10. The objective function of an LPP is 1

A. a constraint

B. a function to be optimised

C. a relation between the variables

D. unbounded region

11. Find the value of $\sin x$ if $\cot^{-1}\left(-\frac{1}{5}\right) = x$. 1

12. Define minor of an element of a determinant. 1

13. Write the cofactors C_{31} of the determinant $\begin{vmatrix} 1 & 3 & 5 \\ -1 & 2 & 3 \\ 2 & 6 & -7 \end{vmatrix}$. 1

14. When is a function $f(x)$ said to be continuous at $x = c$? 1

15. Evaluate : $\int \frac{dx}{3x+2}$. 1

16. State the first fundamental theorem of integral calculus. 1

17. Solve : $\frac{dy}{dx} = y\cos x$ 1

18. Define position vector of a point. 1

19. If the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} + \lambda\hat{k}$ are perpendicular, then find λ. 1

20. What is the region represented by the inequations $x \geq 0, y \geq 0$? 1

21. Define reflexive and transitive relations. 2

22. Show that $f : N \to N$ defined by $f(x) = 2x$ is one one but not onto. 2

23. Prove that $\cos^{-1}(2x-1) = 2\cos^{-1} x, 0 \leq x \leq 1$. 2

24. If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, find A(adjA) without computing adjA. 2

25. Find the least value of k so that the function $f(x) = x^2 + kx + 1$ is strictly increasing on (1, 2). 2

26. Solve the differential equation $(x^2 + 1)\, dy = xydx$ 2

27. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then show that $|\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}$. 2

28. Find the angle between the lines $\vec{r} = 4\hat{i} - \hat{j} + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$ and $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$. 2

29. Prove that if a function f is derivable at a point c, then it is also continuous at that point. 4

30. If $y=(\tan^{-1} x)^2$, show that $(x^2+1)^2\frac{d^2y}{dx^2}+2x(x^2+1)\frac{dy}{dx}=2$ 4

Or

If $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

31. Find by integration, the area of the region under the curve and above the x – axis of $\frac{x^2}{4}+\frac{y^2}{9}=1$ 4

32. Solve the differential equation

$(x^2 - y^2)\,dx + 2xy\,dy = 0$, given that $x = 1$ when $y = 1$. 4

Or

Solve the differential equation

$\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

33. If the vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC, then find the length of the median through A. 4

Or

Using vectors, find the area of the triangle with vertices A(1,1,2), B(2,3,5) and (1, 5, 5).

34. Solve the following LPP graphically :

Maximize $z = 60x + 15y$ subject to the constraints. 4

$x + y \leq 50,$
$3x + y \leq 90,$
$x, y \geq 0$

(Don't use graph paper)

35. Mother, father and son line up at random for a family picture. If E : Son on one end and F : Father in the middle, then find P(E/F). 4

Or

A die is tossed thrice. Find the probability of getting an odd number at least once.

36. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1}, where I is the identity matrix and 0 is zero matrix. 6

Or

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}, where I is the identity matrix.

37. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}\,r$. 6

Or

A tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of water is rising at the instant when the depth of water in the tank is 10 m.

38. Prove that $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$ and hence deduce that 6

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

39. Evaluate $\int \frac{dx}{3x^2 + 13x - 10}$ 6

Or

Prove that $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx = \frac{\pi^2}{4}$.

40. Find the shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s-1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$. 6

Or

Find the vector equation of the line passing through $(1,2,-4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$.

41. An insurance company insured 1500 scooter drivers, 2500 car drivers and 4500 truck drivers. The probability of a scooter, a car and a truck driver meeting an accident is 0.01, 0.02 and 0.04 respectively. If one of the insured person meets with an accident, then find the probability that he is a truck driver. 6
