2022

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. ***1 – 4,*** *write the letter associated with the correct answer.*

1. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is 1

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $-\frac{7}{6}$

D. $\frac{7}{6}$

2. The value of $\int \frac{\log x}{x}\, dx$ is 1

A. $\frac{1}{2}(\log x)^2 + c$

B. $2 \log x + c$

C. $x \log x + c$

D. $2 e^x + c$

3. The angle between the vectors $\hat{i}-\hat{j}$ and $\hat{j}-\hat{k}$ is 1

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

4. If the line $\vec{r}=(\hat{i}-2\hat{j}+\hat{k})+\lambda(2\hat{i}+\hat{j}+2\hat{k})$ is parallel to the plane $\vec{r}.(3\hat{i}-2\hat{j}+m\hat{k})=14$, then the value of m is 1

A. 1

B. -4

C. 3

D. -2

5. Define a symmetric relation. 1

6. Find the value of $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$. 1

7. Define an identity matrix. 1

8. Write the definition of Determinant . 1

9. If $\begin{bmatrix} 1-x & 2 & 3 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$ is a singular matrix, find the value of x. 1

10. State Rolle's Theorem. 1

11. Write the relation between the values of a and b so that the function f defined by

$f(x) = \begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$. 1

12. Find the intervals which the function f given by $f(x) = 2x^2 - 3x$ is increasing. 1

13. Write the formula for integration by parts. 1

14. Define vector product of two vectors. 1

15. Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1,1]$. 2

16. Show that $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$. 2

17. Prove that if any two rows (or columns) of a determinant are interchanged, then the sign of the determinant changes. 2

18. Find the adjoint of a matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. 2

19. Write the steps to solve first order linear differential equation. 2

20. Find the general solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$. 2

21. Find the unit vector perpendicular to both the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$. 2

22. Prove that $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$. 2

23. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. 2

24. Prove that the function $f : R \rightarrow R$ defined as $f(x) = 2x - 3$ is invertible. Also find f^{-1}. 4

25. If $y = x^{\sin x} + (\sin x)^{\cos x}$, then find $\frac{dy}{dx}$. 4

26. Show that the function $f(x) = |x - 1|$, $x \in R$ is continuous but not differentiable at $x = 1$. 4

OR

If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$.

27. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x} dx$. 4

OR

Evaluate : $\int_1^3 \left(x^2 + x\right) dx$ as the limit of sum.

28. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. 4

29. Solve the differential equation $(x^2 - y^2)\, dx + 2xy\, dy = 0$ given that $y = 1$ when $x = 1$. 4

30. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$. 4

31. Find the mean and variance of heads in three tosses of a fair coin. 4

32. Using elementary transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$ 6

33. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area. 6

OR

A jet of an enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3,7), wants to shoot down the jet when it is nearest to him. Find the nearest distance.

34. Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ 6

OR

Evaluate : $\int (x-5)\sqrt{x^2+x}\, dx$

35. Derive the vector equation of a plane in the normal form and hence obtain the cartesian equation of the plane. 6

OR

Derive the vector equation of a line passing through a given point and parallel to a given vector, and hence obtain the cartesian equation of the line.

36. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economy class

than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. Solve the L.P.P. graphically. (No graph paper will be supplied) 6

37. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ? 6
