2017

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three Hours and *Fifteen Minutes

(*15 minutes are given as extra time for reading questions)

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

*For Question Nos. **1-6**, write the letter associated with the correct answer.*

1. The relation R on the set of rational numbers defined by

   $R = \{(x,y) : x, y \in Q \text{ and } x < y\}$ is

   A. reflexive.

   B. symmetric.

   C. transitive.

   D. none of these. 1

2. The principal value of $tan^{-1}\left(tan\frac{35\pi}{3}\right)$ is

A. $\frac{35\pi}{3}$.

B. $\frac{5\pi}{3}$.

C. $\frac{\pi}{3}$.

D. $-\frac{\pi}{3}$. 1

3. If the function $f(x)=\begin{cases}\frac{sin 3x}{4x}, & \text{for } x\neq 0\\ k\ \ , & \text{for } x=0\end{cases}$

is continuous at the point $x=0$, then the value of k is

A. 1.

B. $\frac{3}{4}$.

C. $\frac{4}{3}$.

D. 5. 1

4. The equation of the normal at the point (x_1, y_1) to a curve $y = f(x)$ may be written as :

A. $y - y_1 = \left[\frac{dy}{dx}\right]_{(x_1, y_1)} (x - x_1)$.

B. $x - x_1 = \left[\frac{dy}{dx}\right]_{(x_1, y_1)} (y - y_1)$.

C. $y - y_1 = \frac{x - x_1}{\left[\frac{dy}{dx}\right]_{(x_1, y_1)}}$.

D. $(x - x_1) + \left[\frac{dy}{dx}\right]_{(x_1, y_1)} (y - y_1) = 0$. 1

5. If $\int_0^a 3x^2 dx = 8$, then the value of a is

A. 2.

B. 3.

C. 1.

D. $\frac{3}{2}$. 1

6. The distance of the point with position vector $\vec{a}$ from the plane $\vec{r}.\vec{n} = q$ is

A. $|\vec{a}.\vec{n} - q|$.

B. $\dfrac{|\vec{a}.\vec{n} - q|}{|\vec{n}|}$.

C. $\dfrac{|\vec{a}.\vec{n} - q|}{|\vec{a}|}$.

D. $\dfrac{|\vec{a}.\vec{n} - q|}{|\vec{q}|}$. 1

7. When is a binary operation $*$ on a set A said to be associative ? 1

8. Define transpose of a matrix A. 1

9. Is Lagrange's mean value theorem applicable to the function $f(x) = |x|$ in the interval [–1, 1] ? 1

10. The length and breadth of a rectange are increasing at the rate of $\frac{1}{2} cm/sec$ each. How fast is the perimeter of the rectangle increasing ? 1

11. Write in words the formula of integration by parts. 1

12. Give a standard form of a linear differential equation of first order. 1

13. Define scalar product of two vectors $\vec{a}$ and $\vec{b}$. 1

14. If α, β, γ are the direction angles of a line, show that $sin^2 \alpha + sin^2 \beta + sin^2 \gamma = 2$. 1

15. Write the expression for the shortest distance 'd' between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ in terms of the vectors $\vec{a}_1, \vec{a}_2$, $\vec{b}_1$ and $\vec{b}_2$. 1

16. Define mean or expected value of a discrete random variable. 1

17. Show that the function $f : R \to R$ defined by $f(x) = \frac{2x-1}{3}$, $x \in R$ is one-one and onto. 3

18. Using properties of determinants prove that 3

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

19. Prove that a square matrix A is invertible if and only if A is non-singular. 3

20. Define a differential equation, its order and degree. 3

21. Find the value of λ for which the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$, $\hat{i} - \lambda\hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - 5\hat{k}$ are coplanar. 3

22. If the mean of a binomial distribution is 3 and variance is $\frac{3}{2}$, find the probability of at least 5 successes. 3

23. Prove that $sin^{-1}\frac{4}{5} + sin^{-1}\frac{5}{13} + sin^{-1}\frac{16}{65} = \frac{\pi}{2}$ 4

OR

If $\sin^{-1} x+\sin^{-1} y+\sin^{-1} z=\frac{\pi}{2}$, prove that $x^2+y^2+z^2+2xyz=1$.

24. When is a function said to be derivable at a point? Prove that if a function is derivable at a point, then it is continuous at that point. 4

25. If $(\sin x)^y = x+y$, prove that $\frac{dy}{dx}=\frac{1-(x+y)y\cot x}{(x+y)\log\sin x-1}$. 4

OR

If $\sin^{-1} y=\log x$, prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

26. Draw a rough sketch of the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and find the area enclosed by the curve using integration. 4

27. Solve the differential equation $x^2\,dy+y\,(x+y)\,dx=0$ given that $y=1$ when $x=1$. 4

28. Show that the perpendicular distance of the point $\vec{c}$ from the line joining the points $\vec{a}$ and $\vec{b}$ is

$$\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}.$$ 4

29. Using elementary transformations, find the inverse of the matrix: 6

$$A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

30. Show that the right circular cone of maximum volume which can be inscribed in a sphere of radius a, has its altitude equal to $\frac{4}{3}a$. 6

OR

A square piece of tin of side 18*cm* is to be made into a box without top, by cutting a square each from corners and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is maximum. Find the maximum volume also.

31. Prove that $\int\frac{dx}{\sqrt{a^2-x^2}}=sin^{-1}\frac{x}{a}+C$ and deduce that $\int\sqrt{a^2-x^2}\,dx=\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}sin^{-1}\frac{x}{a}+C$. 6

32. Evaluate $\int_0^1 \frac{log\,(1+x)}{1+x^2}\,dx$. 6

OR

Prove that $\int_0^{\frac{\pi}{2}} log\;\; sin\,x\,dx=-\frac{\pi}{2}log\,2$.

33. Find the image of the point with position vector $\hat{i}-3\hat{k}$ in the line $\vec{r}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$. 6

OR

Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ measured parallel to the plane $3x+2y+2z-5=0$.

34. A housewife wishes to mix two kinds of foods *X* and *Y*, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one *kg* of each food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kilogram of food *X* costs rupees 6 and one kilogram of food *Y* costs rupees 10. Find the least cost of the mixture which will produce the desired diet. Solve the L.P.P. graphically. (Graph paper will not be supplied). 6

35. An urn contains five balls. Two balls drawn at random from the urn are found both to be white. Find the probability that all the five balls in the urn are white. 6