2017

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three Hours and *Fifteen Minutes

(*15 minutes are given as extra time for reading questions)

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. ***1-6****, write the letter associated with the correct answer.*

1. The identity element of the binary operation $*$ on R defined by $a*b=\dfrac{ab}{4},\ \forall\, a, b\in R$ is : 1

   A. 0

   B. 1

   C. 4

   D. 16

2. The function $f(x) = (x+1)^3 (x-3)^3$ is increasing in the interval : 1

A. $(-1, 3)$

B. $(-\infty, 1)$

C. $(1, \infty)$

D. $(-\infty, \infty)$

3. $\int_0^1 \sqrt{1-x^2}\, dx$ equals : 1

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 2π

4. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is : 1

A. π

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. 2π

5. The distance between the planes $2x+3y+4z=4$ and $4x+6y+8z=12$ is : 1

A. 2 units

B. 8 units

C. $\frac{2}{\sqrt{29}}$ units

D. $\frac{8}{\sqrt{29}}$ units

6. If $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$, the angle between $\vec{a}$ and $\vec{b}$ is : 1

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

7. Show that the function $f:R\rightarrow R$ defined by $f(x)=4x-3,\ \forall\ x\in R$ is injective. 1

8. Give geometrical interpretation of Lagrange's Mean Value Theorem. 1

9. Is Rolle's Theorem applicable to the function $f(x)=\tan x$ in the interval $[0,\pi]$? 1

10. Find the slope of the normal to the curve $y=3x^2-\sin x$ at $x=0$. 1

11. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ 1

12. What is meant by a solution of a differential equation? 1

13. Define position vector of a point. 1

14. If α, β, γ be the angles made by a line with the coordinate axes, prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$. 1

15. Find the angle between the lines whose direction ratios are (1,–2, 3) and (1,–1,–1). 1

16. If $P(B)=\frac{1}{3}$ and $P(A/B)=\frac{2}{5}$, find $P(A\cap B)$. 1

17. Prove that the relation R defined in $\mathbb{Z}$ by xRy if and only if $|x-y|$ is a multiple of $5(x,\ y\in\mathbb{Z})$ is an equivalence relation. 3

18. If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, prove that $A^3=A^{-1}$. 3

19. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx$. 3

20. Prove that $\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+C$ 3

21. Find the differential equation of the family of curves $y = e^x (A\cos x + B\sin x)$, where A and B are arbitrary constants. 3

22. A bag contains 3 black and 2 white balls and another bag contains 2 black and 4 white balls. One bag is chosen at random and from it a ball is drawn. Find the probability that the ball drawn is white. 3

23. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$ 4

Or

If $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, prove that

$$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$$

24. If the inverse of a square matrix exists, prove that it is unique. If A and B are both invertible square matrices of the same order, prove that $(AB)^{-1} = B^{-1}A^{-1}$. 4

25. If $x\sin(a+y) + \sin a\cos(a+y) = 0$, prove that 4

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Or

Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

26. If a function f is differentiable at a point, prove that it is also continuous at that point. 4

27. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$. 4

Or

Find the area of the region bounded by the lines $x+2y=2$, $y-x=1$ and $2x+y=7$.

28. Find the integrating factor of the linear equation $\frac{dy}{dx}Py=Q$ and hence obtain the general solution of the equation. 4

29. Using the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$, solve the following system of equations : 6

$$x-y+2z=1$$
$$2y-3z=1$$
$$3x-2y+4z=2$$

30. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. 6

Or

Prove that the curves $y^2=x$ and $xy=k$ cut at right angles if $8k^2=1$.

31. Prove that : 6

$$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}$$

Or

$$\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{\sqrt{a^2+b^2}} \cos\left(bx - \tan^{-1}\frac{b}{a}\right) + C$$

32. Define cross product of two vectors and give the geometrical interpretation of the cross product of two vectors. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, obtain the algebraic formula for $\vec{a} \times \vec{b}$. 6

Or

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, obtain the algebraic formula for the scalar triple product $\vec{a}.\left(\vec{b}\times\vec{c}\right)$, and hence prove that $\vec{a}.\left(\vec{b}\times\vec{c}\right) = \vec{b}.\left(\vec{c}\times\vec{a}\right) = \vec{c}.\left(\vec{a}\times\vec{b}\right)$.

33. Derive the vector equation of a line passing through two given points and hence obtain the Cartesian equation of the line. 6

Or

Derive the vector equation of a plane passing through three non-collinear points and hence obtain the equation of a plane in the intercept form.

34. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the probability distribution, mean and variance of the number of aces. 6

35. Two go-downs A and B have a grain storage capacity of 100 quintals and 50 quintals respectively. They supply grain to three ration shops D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from go-downs to the shops are given in the table below :

To / From	Transportation cost per quintal (in rupees)		
	D	E	F
A	6	3	2.50
B	4	2	3

How should the supplies be transported in order that the transportation cost is minimum ? Solve the problem graphically. 6
